Exact and Asymptotic Solutions to Korteweg-de Vries Equation

Valerii Samoilenko
Taras Shevchenko National University of Kyiv, Kyiv, Ukraine
e-mail: valsamyul@gmail.com
Yuliia Samoilenko
Taras Shevchenko National University of Kyiv, Kyiv, Ukraine
e-mail: yusam@univ.kiev.ua

We consider Korteweg-de Vries equation being one of the fundamental object of modern mathematical and theoretical physics and possessing a lot of different interesting properties. It attracted much attention of scientists in past century after discovery of new properties of its solitary wave solutions in 1965.

Our reasoning start with problem on exact solutions to the Korteweg-de Vries equation with constant coefficients as well as their some specific properties.

Later we consider the equation with small regular and singular perturbations, in particular, a problem on constructing its asymptotic solutions. In this connection we note contribution of R. Miura, M. Kruskal, D. de Kerf, V.P. Maslov, S.Yu. Dobrokhotov, G.A. Omelyanov, A.M.Ilyin, S.G. Glebov, A.M. Kiselev, V.A. Lazarev ant others to studying the problem.

Finally we present our results on constructing asymptotic soliton type solutions to singularly perturbed equation with variable coefficients

$$\varepsilon^n u_{xxx} = a(x,t,\varepsilon)u_t + b(x,t,\varepsilon)uu_x, \quad n \in \mathbf{N}, \tag{1}$$

where functions $a(x,t,\varepsilon)$, $b(x,t,\varepsilon)$ are written in the form of asymptotic series

$$a(x,t,\varepsilon) = \sum_{k=0}^{N} \varepsilon^k a_k(x,t) + O(\varepsilon^{N+1}),\ b(x,t,\varepsilon) = \sum_{k=0}^{N} \varepsilon^k b_k(x,t) + O(\varepsilon^{N+1})$$

as well as results on constructing the asymptotic solutions of Cauchy problem to equation (1) with initial condition of the following form $u(x,0,\varepsilon) = f(x,\varepsilon)$, where $f(x,\varepsilon) \in C^\infty(\mathbf{R})$ or $f(x,\varepsilon) \in \mathcal{S}(\mathbf{R})$, $x \in \mathbf{R}$. Here ε is a small parameter.

We propose the algorithm of constructing asymptotic soliton type solutions of equation (1) and Cauchy problem for it.

The algorithm is developed for finding asymptotic one-, two- and multy-phase soliton type solutions to these problems.

It should be noted that structure of these asymptotic solutions essentially depends on the degree of small parameter at the highest derivative in (1).

We also study question on accuracy with which constructed asymptotic solutions satisfy the problems under consideration. In particular, theorems on asymptotic estimates for constructed solutions are obtained.

References

[1] V. Hr. Samoylenko, Yu. I. Samoylenko, "Asymptotic multiphase soliton-like solutions of the Cauchy problem for a singularly perturbed Korteweg–de-Vries equation with variable coefficients", *Ukr. Math. J.*, **66**, No. 12, 1842 – 1861 (2015).